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SHORTER COMMUNICATIONS

AN ANALYSIS FOR LOW TURBULENCE INTENSITY LIQUID METAL HEAT TRANSFER

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INTRODUCTION

TURBULENT heat transfer analyses for liquid metals, as well as other fluids, have generally been tied to the classical eddy diffusivity concept. However, an alternative approach to the analysis of heat transfer to fully developed turbulent tube flow of liquid metals has recently been offered which is based on the surface renewal and penetration concept [11, 12]. This formulation suggests that the mean turbulent heat transfer flux is a function of (1) the energy transport to eddies in transit from the turbulent core to the wall region, (2) the molecular transport to fluid within the wall region, and (3) the mean frequency of renewal. Accordingly, the simple one dimensional unsteady conduction equation,

$$\frac{\partial t}{\partial \theta} = \alpha \frac{\partial^2 t}{\partial y^2} \tag{1}$$

is coupled with initial and boundary conditions of the form $t(0, y) = T_i$, $t(\theta, \infty) = T_i$, and $t(\theta, 0) = T_0$ (for uniform wall temperature, T_0), or $\partial t(\theta, 0)/\partial y = -g_0/k$ (for uniform wall flux, q_0); t is the instantaneous temperature within individual elements of fluid at the wall, θ is the instantaneous contact time, y is this distance from the wall and α is the thermal diffusivity. The solution of this system of equations and the use of the age distribution principle (using Danckwert's [3] random contact time distribution) gives rise to expression for the local mean Nusselt number, hD/k, of the form

$$Nu = \frac{T_0 - T_i}{T_0 - T_b} \frac{D}{\sqrt{(\alpha\tau)}} \tag{2}$$

where T_i is the eddy temperature at the first instant of renewal, T_b is the bulk stream temperature and D is the tube diameter. The mean frequency of renewal is equivalent to the inverse of the mean residence time, τ. A reasonable formulation for τ has been obtained on the basis of the adaptation of the surface renewal and penetration model to momentum transfer which leads to an expression of the form [11]

$$U^*\sqrt{\left(\frac{\tau}{\nu}\right)} = \sqrt{\left(\frac{2}{f}\right)} \tag{3}$$

where f is the Fanning friction factor, $U^* = \sqrt{(f/2)}\ U$, U is the bulk stream velocity, and ν is the kinematic viscosity.

In other formulations of the surface renewal and penetration model the parameter T_i has generally been set equal to the bulk stream temperature, T_b. This assumption appears to be reasonable for fluids with large values of the Prandtl number, Pr. However, T_i may not be set equal to T_b for liquid metals due to the significance of molecular transfer. Accordingly, an expression for T_i has been proposed on the basis of an analysis of the energy and momentum transfer to eddies in transit from the turbulent core to the wall region [12]. The coupling of this analysis and the basic surface renewal and penetration model gives rise to an expression for the mean Nusselt number for both uniform wall temperature and uniform wall heat flux boundary conditions of the form

$$Nu = \frac{1}{1 + \psi} \frac{f}{2} Re \sqrt{Pr} \tag{4}$$

where ψ is defined in [12] and Re is the Reynolds number. This expression has been shown to compare favorably with experimental heat-transfer data for fluids with low to moderate values of the Prandtl number and for values of the Péclét number ($Pe = Pr . Re$) greater than 5×10^3. Equation (4) is now compared with experimental heat-transfer data for liquid metals (wetting systems) for a larger Péclét number range in Fig. 1. This expression seriously diverges from the data for values of the Péclét number much less than 5×10^3.

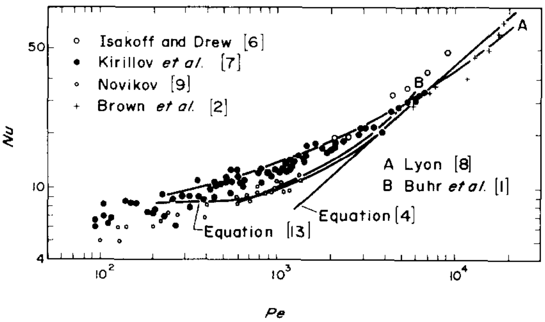

FIG. 1. Comparison of present analysis with experimental data for turbulent flow in tubes with uniform wall heat flux

A MODIFICATION FOR LOW TURBULENCE INTENSITY LIQUID METAL FLOW

The elementary surface renewal and penetration model is based on the premise that fluid elements in the close vicinity of the surface may be considered as being semi-infinite in extent i.e. $t(\theta, \infty) = T_i$. In essence, this boundary condition isolates the molecular transport process to eddies, during their residency in the wall region, from the core region. That is, the transport processes within the wall and core regions are said to be singly coupled by the renewal process. This fairly simple representation of the turbulent heat transfer process appears to be adequate for fluids with values of the Péclét number above 5×10^3. However, it is suspicioned that the failure to account for molecular interaction between the wall and core regions becomes increasingly important for decreasing values of the Péclét number.

The influence on the heat transfer process within the wall region exerted by the overlying fluid may be simulated by assuming that this thoroughly mixed region imposes a strata of fluid at some distance δ which is essentially maintained at temperature T_δ. Accordingly, it is now suggested that the boundary condition away from the wall be replaced by

$$t = T_\delta \quad \text{at } y = \delta \tag{5}$$

where the temperature T_δ is assumed to be intermediate between T_0 and T_b. The inclusion of this boundary condition in the above analysis gives rise to an expression for the local mean Nusselt number of the form

$$Nu = \frac{D}{\sqrt{(\alpha\tau)}} \coth\left(\frac{\delta}{\sqrt{(\alpha\tau)}}\right)\left[\frac{T_0 - T_i}{T_0 - T_b} + \left(\frac{T_i - T_\delta}{T_0 - T_b}\right)\left\{\cosh\left(\frac{\delta}{\sqrt{(\alpha\tau)}}\right)\right\}^{-1}\right]. \tag{6}$$

In an attempt to evaluate the proposed model, reasonable assumptions regarding the parameters δ and T_δ must be made.

A formulation for δ

Experimental evidence suggests that turbulent heat transfer to liquid metals at values of the Péclét number of the order of 100 may be characterized as simple slug flow. Significantly, this information may be used in conjunction with equation (6) to obtain an expression for δ. The limiting value of the Nusselt number given by equation (6) for decreasing values of $\delta/\sqrt{(\alpha\tau)}$ becomes

$$\lim_{\delta/\sqrt{(\alpha\tau)} \to 0} Nu = Y = \frac{T_0 - T_\delta}{T_0 - T_b}\frac{D}{\delta}. \tag{7}$$

Hence, a simple expression may be written for δ as

$$\delta = \frac{D}{Y}\frac{T_0 - T_\delta}{T_0 - T_b}. \tag{8}$$

The limiting value of the Nusselt number for slug flow may be obtained from the analysis by Lyon [8] for uniform heat flux ($Y = 8{\cdot}0$) and Seban and Shimazaki [10] for uniform wall temperature ($Y = 6{\cdot}0$).

Further insight into the physical significance of the parameter δ can be gained from a consideration of the mean thermal penetration depth, Δ, associated with the solution of equation (1). An approximate solution to the synonymous integral energy equation leads to an expression of the form

$$\Delta = \frac{\sqrt{\pi}}{2}\sqrt{\left(\frac{40}{3}\right)}\sqrt{(\alpha\tau)} \tag{9}$$

for uniform wall temperature, and

$$\Delta = \frac{\sqrt{\pi}}{2}\sqrt{\left(\frac{20}{3}\right)}\sqrt{(\alpha\tau)} \tag{10}$$

for uniform wall flux. With τ given by equation (3), an expression for Δ may be written as

$$\frac{\Delta}{D} = \frac{\sqrt{\pi}}{2}\sqrt{\left(\frac{40}{3}\right)}\frac{\sqrt{Pr}}{f/2\,Pe} \tag{11}$$

for uniform wall temperature. As mentioned, the thermal penetration depth appears to become significant for values of the Péclét number of the order of 5×10^3. Hence, δ may be assumed to be approximately equal to the value of Δ at $Pe = 5 \times 10^3$; equation (11) gives a value of Δ/D equal to $0{\cdot}0628$ for $Pe = 5 \times 10^3$ and $Pr = 0{\cdot}05$. Based on the assumption that T_δ is approximately equal to T_i, equation (8) gives rise to a value or δ/D equal to $0{\cdot}0834$; the agreement between these two values is satisfactory. Interestingly, the ratio of the mean thermal penetration depths for uniform wall temperature and uniform wall flux is equal to $\sqrt{2}$, which is in basic agreement with the ratio of the limiting values of the Nusselt number for these two boundary conditions, 1·33.

An approximation for T_δ

Based on equations (6) and (8), an expression may be written for the mean Nusselt number of the form

$$Nu = A \coth \frac{A + X}{Y} + \frac{X}{\sinh [(A + X)/Y]} \tag{12}$$

where

$$A = \frac{T_0 - T_i}{T_0 - T_b} \frac{D}{\sqrt{(\alpha\tau)}}; \qquad X = \frac{T_i - T_\delta}{T_0 - T_b} \frac{D}{\sqrt{(\alpha\tau)}}.$$

With τ defined by equation (3), A and X become

$$A = \frac{1}{1+\psi} \frac{f}{2} Re \sqrt{Pr}; \qquad X = \frac{T_i - T_\delta}{T_0 - T_b} \frac{f}{2} Re \sqrt{Pr}.$$

It may be assumed that fluid overlying the wall region is frequently refreshed by eddies from the bulk stream which do not enter the wall region itself. Hence, the temperature T_δ apparently lies between T_i and T_b. Computations for the mean Nusselt number obtained on the basis of these expressions are quite insensitive to values of T_δ in this range such that T_δ may be set equal to T_i without seriously affecting the results.

DISCUSSION

With T_δ set equal to T_i, equation (12) takes the form

$$Nu = A \coth \frac{A}{Y}. \tag{13}$$

This expression is compared with experimental heat transfer data for uniform wall heat flux ($Y = 8{\cdot}0$) in Fig. 1. A recent correlation by Buhr *et al.* [1] is also shown which is based on a consideration of the effects of free convection. The calculations for the Nusselt number based on equation (13) essentially fall on a single curve for fluids with values of the Prandtl number in the liquid metal range. Equation (13) is seen to follow the data trend remarkably well and is in basic agreement with the correlation of Buhr *et al.* It appears, however, that a final firm judgement regarding the validity of the proposed model and the available data can only be made after much more experimental evidence becomes available.

CONCLUDING REMARKS

The present analysis demonstrates the potential usefulness of the surface renewal principle in characterizing turbulent heat transfer to liquid metals. Although the attempt to account for the interaction between the turbulent core and the wall region by the use of equation (5) is somewhat primitive, this analysis appears to be fairly representative of the actual process for low turbulence intensity.

Interestingly, Toor and Marchello [13] have proposed a similar model, known as the film–penetration model, in which T_i has been set equal to T_b. Equation (12) reduces to their result for $T_i = T_\delta = T_b$, i.e. (in terms of δ)

$$Nu = \frac{D}{\sqrt{(\alpha\tau)}} \coth \frac{\delta}{\sqrt{(\alpha\tau)}}. \tag{14}$$

These workers have suggested that the film-penetration model is representative of the turbulent heat (mass) transfer to fluids with large values of the Prandtl number for turbulent flow in tubes. (The film-penetration model has also been adapted to mass transfer at a fluid–fluid interface by Dobbins [4].) However, the present analysis suggests that the effect on the molecular transfer within the wall region of the overlying fluid diminishes with increasing values on the Prandtl number, such that this type model is not useful for fluids of high Prandtl number.

ACKNOWLEDGEMENT

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A DIAMETRAL EFFECT ON VAPOUR COLUMN FORMATIONS IN FILM BOILING IN CARBON DIOXIDE NEAR THE CRITICAL STATE

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1. INTRODUCTION

IN A RECENT article [1] conclusions were drawn and interesting photographic evidence produced concerning the phenomena of uprising vapour in columns. The present communication describes early visual observations of an ongoing investigation, and the authors feel obliged to comment on the conflicting evidence of the diametral effect on the dominant Taylor wavelength λ, which is the measured distance between rising vapour columns in film boiling.

2. EXPERIMENT

The present note reports on one geometry, namely a horizontal stainless steel tube, arranged in a cylindrical test chamber which incorporates glass windows so situated as to enable flow visualization. The optical arrangement adopted for the schlieren type experiment is shown in Fig. 1. Light is transmitted from a simple filament source through a collimating lens, to provide a light beam which is transmitted through the fluid and a second lens, and finally on to a knife edge positioned to eliminate the direct image of the source; the desired image is focussed in the camera.

Carbon dioxide is supplied from commercial bottles, equipped with a syphon tube, and warm water is passed through passages in the test vessel until the pressure reaches some predetermined level. A standard Bourdon type test gauge is used to measure gauge pressure, and the bulk temperature is measured by a number of chromel/alumel thermocouples. The tube surface temperature is determined by monitoring the tube internal temperature history using fine chromel/alumel thermocouples, and computing the external surface temperature in both radial and axial directions. The thermocouple outputs are recorded on a Tinsley potentiometer.

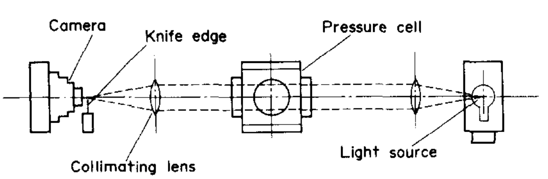

FIG. 1. Simple Schlieren optical arrangement.

3. RESULTS

It is well known that film boiling in the critical region results in various characteristic shapes of the uprising vapour: bubbles, columns and sheets, and that the characteristic Taylor wavelength is strongly dependent on the pressure and wire diameter. The authors wish to draw attention to the latter effect. The photograph, Fig. 2. shows vapour column formations in bulk conditions of 0·98 reduced pressure and 30·5°C on a stainless steel tube 1·07 mm dia., it is regretted that the heat flow data were not available. The significant feature is the uniformity of the column spacing λ, which appears to contradict the observations of [1] in which it was concluded that the observed column spacing was quite uniform for the smaller 0·1 mm dia. wire and irregular for the largest, 0·38 mm wire. The same uniformity as exemplified in Fig. 2 was achieved at various other bulk conditions. and the formation of the columns followed closely Zuber's idealized model in which columns of vapour, formed by the coalescence of a large number of